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Airfoil Response to an Incompressible Skewed Gust of Small Spanwise Wave-Number

R. K. Amiet*
United Technologies Research Center, East Hartford, Conn.

Nomenclature

b	= semichord
C_p	= pressure coefficient
$f(M)$	= defined by Eq. (4)
$g(\xi)$	= defined by Eq. (7)
k_x	$=\omega b/U$
k_y	$=2\pi b/$spanwise gust wavelength
L	= lift normalized by $2\pi\rho_0 U w_g b e^{i(\omega t - k_{y0}y)}$
Ω	= Mach number of freestream
P	= pressure
S	= Sears function
t	= time
U	= freestream velocity
w_g	= gust upwash
x,y	= chordwise and spanwise coordinates normalized by b
β^2	$=1-M^2$
σ	$=Mk_x/\beta k_y$
ω	= circular frequency

Subscripts

0	= quantity related to incompressible skewed gust problem
∞	= quantity related to compressible parallel gust problem

Introduction

THE present Note derives an approximate solution for the response function of an infinite span airfoil in a three-dimensional (the gust wave fronts skewed relative to the airfoil leading edge) gust convecting with the freestream incompressible flow. The solution is limited to small spanwise wave number k_y. Together with the large k_y solution[1] (see also Ref. 2), the entire range of k_y can be observed. The present solution is derived from the parallel compressible gust result of Amiet,[3] using the similarity results of Graham.[4]

For the problem of an airfoil in a gust convecting with the freestream, Graham[4] related the general case of a skewed gust in compressible flow to either of the simpler cases of a parallel gust in compressible flow or a skewed gust in incompressible flow. If the parameter $\sigma \equiv Mk_x/\beta k_y \geq 1$, the similarity is to the parallel compressible gust case, whereas, if $\sigma \leq 1$, the similarity is to the skewed incompressible gust case. Thus, the solution for the parallel compressible gust case can be used throughout the regime $\sigma \geq 1$, and the skewed incompressible gust solution can be used in the regime $\sigma \leq 1$.

It was pointed out by Graham,[4] although it does not appear to be known widely, that each of these similarity rules can be used outside the regime for which it primarily is applicable. Thus, the solution for the case $\sigma < 1$ can be generated from the solution for $\sigma > 1$, and in particular, the result for a skewed incompressible gust ($\sigma = 0$) can be derived. The similarity rule used in the present note to relate the skewed incompressible gust case to the parallel compressible gust case was derived first by Graham, who gives further discussion and proof of the rule.[4]

Other approximate solutions to the problem of an airfoil encountering an incompressible skewed gust have been presented.[5,6] That of Ref. 5 is also a small k_y solution, although the order of accuracy was not given. The approximate two-dimensional compressible solution used as the base for the present solution was shown by Amiet,[3] and formally proven by Kemp and Homicz,[7] to be accurate to $O(Mk_x/\beta^2)$. From this, the skewed gust result presented herein is shown to be accurate to $O(k_y)$. Reference 6 gives an approximate solution which can be applied to all k_y. However, the present small k_y solution, together with the large k_y solution given in Refs. 1 and 2, gives significantly improved accuracy.

Result

The solution given by Graham[4] for the relation between the pressure coefficients C_p for the skewed incompressible gust case (subscript 0) and the parallel compressible gust case (subscript ∞) is

$$C_{p0}(x,k_{x0},k_{y0}) = \beta_\infty C_{p\infty}(x,k_{x\infty},M_\infty) \times \exp(-ik_{x\infty}M_\infty^2 x/\beta_\infty^2 - ik_{y0}y) \tag{1}$$

where

$$k_{x0} = k_{x\infty}/\beta_\infty^2 \qquad k_{y0}/k_{x0} = i\,M_\infty \tag{2}$$

In order to find the response to an incompressible skewed gust with small k_{y0}, the solution of Amiet[3] for the case of a low-frequency parallel compressible gust will be utilized. This solution has for the airfoil surface pressure,

$$P_\infty(x,t,k_{x\infty},M_\infty) = \mp(\rho_0 U w_g/\beta_\infty) \times [(1-x)/(1+x)]^{1/2} S(k_{x\infty}/\beta_\infty^2) \exp(i\{\omega t + k_{x\infty}[xM_\infty^2 + f(M_\infty)]/\beta_\infty^2\}) \tag{3}$$

where

$$f(M_\infty) = (1-\beta_\infty)\ln M_\infty + \beta_\infty \ln(1+\beta_\infty) - \ln 2 \tag{4}$$

and S is the classical Sears function. The minus or plus sign refers to the upper and lower airfoil surfaces, respectively. On substitution of Eqs. (3) and (4) into Eq. (1) and replacing the ∞ variables, using Eqs. (2), the airfoil surface pressure produced by a gust of the form

$$w_g(x,y,t) = w_g \exp[i(\omega t - k_{x0}x - k_{y0}y)] \tag{5}$$

in incompressible flow is found to be

$$P_0(x,y,t,k_{x0},k_{y0}) = P_\infty(x,t,k_{x0},0) \times \exp[ik_{x0}g(k_{y0}/k_{x0}) - ik_{y0}y] \tag{6}$$

where

$$g(\xi) = [(1+\xi^2)^{1/2} - 1]\,[i(\pi/2) - \ln\xi)] + (1+\xi^2)^{1/2}\ln[1 + (1+\xi^2)^{1/2}] - \ln 2 \tag{7}$$

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*Senior Research Engineer. Member AIAA.

Table 1 Approximate solution compared with Graham's results

		Equation (8)		Graham[8]	
k_{xo}	k_{yo}	Amplitude	Phase	Amplitude	Phase
0	0	1	0	1	0
0	0.1	0.8546	0		
0	0.25	0.6752	0	0.6762	0
0	0.5	0.4559	0	0.4869	0
0	1.0	0.2079	0	0.2970	0
0.25	0	0.6744	−12.3	0.6744	−12.3
0.25	0.1	0.6543	−10.0		
0.25	0.25	0.5732	−4.4	0.5930	−4.9
0.25	0.5	0.4151	3.1	0.4623	1.6
0.25	1.0	0.1978	12.2	0.2925	7.4
0.5	0	0.5265	−4.8	0.5265	−4.8
0.5	0.1	0.5184	−3.2		
0.5	0.25	0.4799	1.7	0.4899	0.2
0.5	0.5	0.3803	11.1	0.4139	7.0
0.5	1.0	0.1994	26.0	0.2809	15.5
1.0	0	0.3896	18.9	0.3896	18.9
1.0	0.1	0.3865	19.9		
1.0	0.25	0.3712	23.4	0.3706	21.6
1.0	0.5	0.3236	31.9	0.3322	26.3
1.0	1.0	0.2032	50.6	0.2499	34.9
2.0	0	0.2801	73.1	0.2801	73.1
2.0	0.1	0.2790	73.7		
2.0	0.25	0.2734	76.0	~0.268	~ 76.0
2.0	0.5	0.2543	82.2	0.2460	77.1
2.0	1.0	0.1933	99.2	0.1989	83.0

and P_∞ is given by Eq. (3), with $M_\infty = 0$, i.e., the Sears gust solution. Thus, for incompressible flow the change in the airfoil response produced by skewing the gust is to multiply the nonskewed result by the factor $\exp\ [ik_{x0}g(k_{y0})/k_{x0})]$ when k_{y0} is small.

Since gk_{x0} is independent of x, the relations for lift and moment also are multiplied by the same factor. Thus, the normalized lift is

$$\tilde{L} = S(k_{x0})\exp[ik_{x0}g(k_{y0}/k_{x0})] \tag{8}$$

Since the quarter-chord moment for the Sears gust case is zero, it is zero for the present solution also. According to the numerical results of Graham,[8] and also the approximate solution for large k_{y0} given in Refs. 1 and 2, the center of pressure does move forward from the quarter-chord point as k_{y0} increases. For small k_{y0}, however, this effect is not particularly important. The accuracy of the parallel gust result, Eq. (3), is $O(k_{x\infty}M_\infty/\beta_\infty^2)$. Thus, from Eqs. (2) the accuracy of the present skewed gust solution is $O(k_{y0})$. The forward movement of the center of pressure evidently must be $O(k_{y0}^2)$ or higher, since it is not given by the present solution.

Values calculated using Eq. (8) are compared in Table 1 with the numerical results of Graham.[8] The present approximate solution agrees favorably with the numerical results, especially considering the fact that the solution for large k_{y0} is shown in Ref. 2 to give accurate results for $k_{y0} > 0.25$. Thus, only the range $0 < k_{y0} < 0.25$ need be covered by the present solution.

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Technical Comments

Comment on "On Transient Cylindrical Surface Heat Flux Predicted from Interior Temperature Response"

Murray Imber*
Polytechnic Institute of New York, Brooklyn, N.Y.

RECENTLY, Chen and Thomsen[1] published a note which purportedly presented an alternative solution for the inverse problem in hollow cylinders. Since the method is based upon the temperature inversion measured by *only one interior probe,* whereas previous investigations[3-4] required two imputs, I believe that a number of comments are in order. Due to the form of the differential equation, the linear inverse problem requires a minimum of three inputs: the initial condition and two others. The latter represent the experimental data taken from embedded sensors which can be either: temperature traces from the thermocouples positioned at two different locations, temperature and heat flux information gathered at one location, or the temperature data coupled with a boundary condition dictated by geometrical-thermal considerations. For the latter, Sparrow[2] derives an inversion method for a cylinder, which is solid, hence the technique is applicable for the thermally symmetric situation. In the hollow cylinder configuration, there is generally no thermal symmetry; consequently another method of analysis must be considered. Imber[4] presents a general extrapolation method for this important experimental situation, which utilizes only the experimental data from two thermocouples. Due to the presence of the modified Bessel functions, a short time solution is generated for temperature prediction in any direction, and numerical results are presented to demonstrate the effectiveness of the procedure.

It should be pointed out that Chen's analytical approach is remarkably similar to Imber's, since it incorporates his concept of exponential alteration or suppression (see Eq. (10)) as well as the methodology for the derivation of the solution. Furthermore, the use of Eq. (10) can present difficulties that the authors should be aware of. For small time, the values for the higher order repeated error integral functions decrease rapidly, accordingly the matrix used to generate the coefficients b_n can be nearly singular when the number of

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*Professor, Mechanical Engineering Department, Polytechnic Institute of New York. Associate Fellow AIAA.